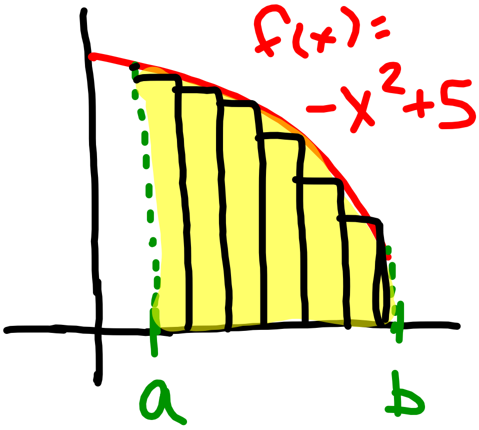
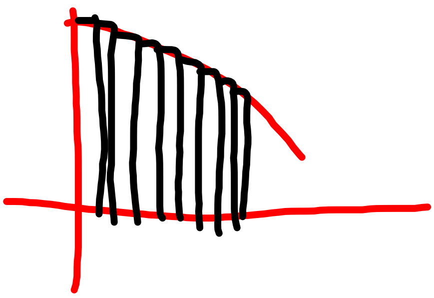
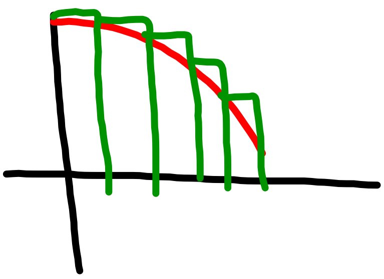
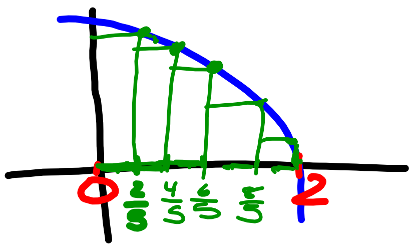
Area in a Plane Region

$f(x) = -x^2+5$ $[a, b]$









$f(x) = -x^2 + 5$

$[0, 2]$

Right Endpoint: $\frac{2}{5} i$

i = # of the rectangle

width height

$$\sum_{i=1}^{5} \frac{2}{5} \left[f\left(\frac{2}{5} i\right) \right]$$

$$\sum_{1}^{5} \frac{2}{5} \left[-\left(\frac{2}{5} i\right)^2 + 5 \right]$$

$$\frac{2}{5} \left[\sum_{1}^{5} \left(-\frac{4}{25} i^2 + 5 \right) \right]$$

$$\frac{2}{5} \left[\sum_{1}^{5} -\frac{4}{25} i^2 + \sum_{1}^{5} 5 \right]$$

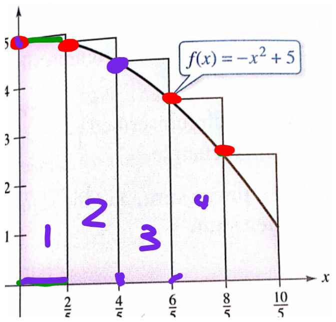
$$\frac{2}{5} \left[-\frac{4}{25} \sum_{1}^{5} i^2 + \sum_{1}^{5} 5 \right]$$

$$\frac{n(n+1)(2n+1)}{6}$$

$$\frac{2}{5} \left(\frac{-4}{25} \cdot \frac{5(5+1)(2(5)+1)}{6} + 25 \right]$$

$$\boxed{6.48}$$

area $[0, 2] > 6.48$



Left Endpoints:

$\frac{2}{5}(i-1)$

$$\sum_{i=1}^{5} \frac{2}{5}\left[f\left(\frac{2}{5}(i-1)\right)\right]$$

$$\frac{2}{5}\left[\sum_{1}^{5} -\left(\frac{2}{5}(i-1)\right)^2 + 5\right]$$

$$\frac{2}{5}\left[\sum_{1}^{5} -\left(\frac{2i-2}{5}\right)^2 + 5\right]$$

$$\frac{2}{5}\left[\sum_{1}^{5} \frac{-4i^2+8i-4}{25} + 5\right]$$

$$\frac{2}{5}\left[\sum_{1}^{5} \frac{-4i^2+8i-4}{25} + \sum_{1}^{5} 5\right]$$

$$\frac{2}{5}\left[\sum_{1}^{5} \frac{-4(i^2-2i+1)}{25} + \sum_{1}^{5} 5\right]$$

$$\frac{2}{5}\left[\frac{-4}{25}\sum_{1}^{5} i^2-2i+1 + \sum_{1}^{5} 5\right]$$

$$\frac{2}{5}\left[\frac{-4}{25}\left(\underbrace{\sum_{1}^{5} i^2}_{\frac{n(n+1)(2n+1)}{6}} - 2\underbrace{\sum_{1}^{5} i}_{\frac{n(n+1)}{2}} + \sum_{1}^{5} 1\right) + \sum_{1}^{5} 5\right]$$

$$\frac{2}{5}\left[\frac{-4}{25}\left(\frac{5(5+1)(2(5)+1)}{6} - 2\left(\frac{5(5+1)}{2}\right) + 5\right) + 25\right]$$

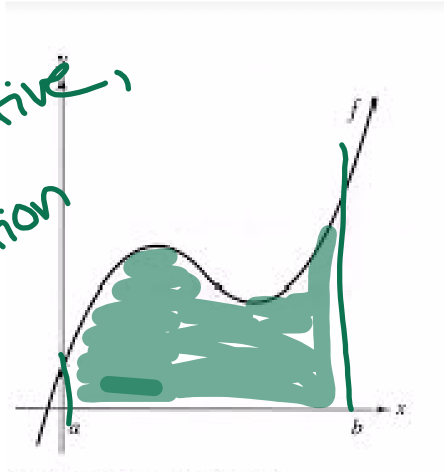
8.08

area

$6.48 < a < 8.08$

non-negative, continuous function

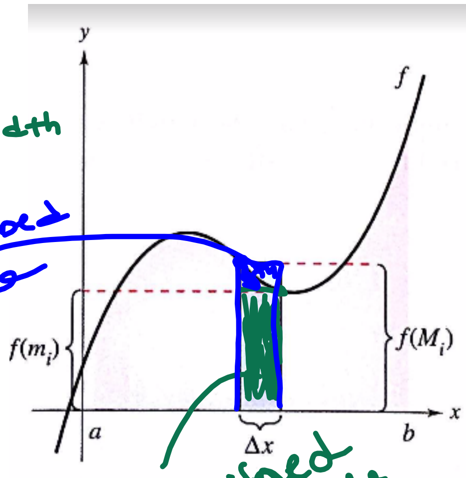
[a, b]



width $\longrightarrow$ $\Delta x = \dfrac{b-a}{n}$ $\leftarrow$ total # of rectangles

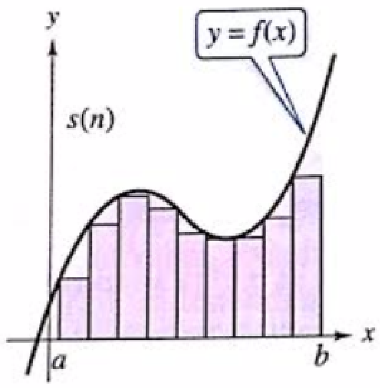
Lower sum= $s(n) = \sum_{i=1}^{n} \underbrace{f(m_i)}_{\text{height}} \underbrace{\Delta x}_{\text{width}}$ ($n \leftarrow$ # of rectangles)

circumscribed rectangle

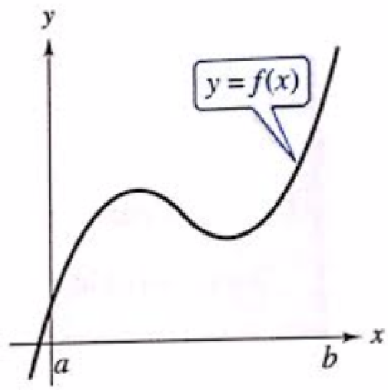


inscribed rectangle

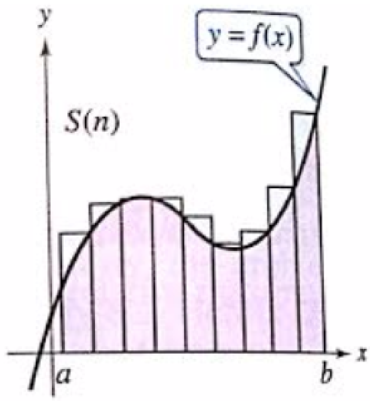
Upper sum= $S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$



Area of inscribed rectangles is less than area of region.

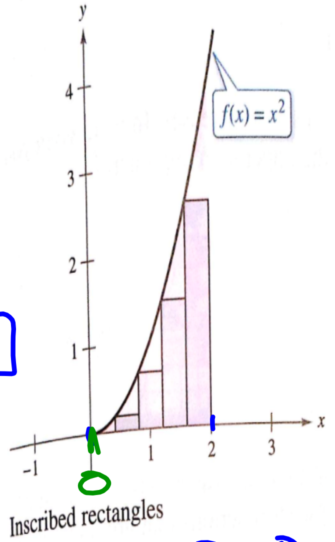


Area of region

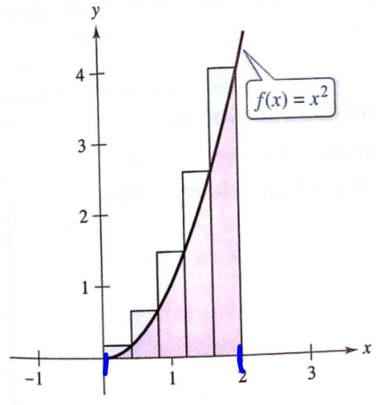


Area of circumscribed rectangles is greater than area of region.

$[0, 2]$



Inscribed rectangles



Circumscribed rectangles

$$\Delta x = \frac{2-0}{n}$$

Left Endpoint

$$m_i = 0 + \frac{2}{n}(i-1)$$

$$= \frac{2i-2}{n}$$

$$\sum_1^n \left(\frac{2}{n}\right)\left(f\left(\frac{2i-2}{n}\right)\right)$$

$$\sum_1^n \frac{2}{n}\left(\left(\frac{2i-2}{n}\right)^2\right)$$

$$\sum_1^n \frac{2}{n}\left(\frac{4i^2-8i+4}{n^2}\right)$$

$$\sum_1^n \frac{2}{n}\left(\frac{4(i^2-2i+1)}{n^2}\right)$$

$$\frac{8}{n^3}\sum_{i=1}^n i^2-2i+1$$

$$\frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6} - 2\left(\frac{n(n+1)}{2}\right) + n\right)$$

Left
$$= \frac{8}{3} - \frac{4}{n} + \frac{4}{3n^2}$$

Right Endpoint